

# CHIRAL PERTURBATION THEORY AND KAON PHYSICS\*

Gerhard Ecker

Inst. Theor. Physik, Univ. Wien  
Boltzmanng. 5, A-1090 Wien, Austria

## Abstract

A brief survey of applications of chiral perturbation theory to both semileptonic and nonleptonic kaon decays is presented. Special emphasis is given to recent theoretical advances related to semileptonic decays, in particular pion pion scattering and  $K_{e4}$  decays. The systematic approach for including isospin violation and electromagnetic corrections in semileptonic kaon decays is discussed for  $K_{l3}$  decays.

To appear in the Proceedings of  
KAON 2001, Pisa, Italy, June 12 - 17, 2001

\* Work supported in part by TMR, EC-Contract No. ERBFMRX-CT980169 (EURODAΦNE).

Table 1: *From the Fermi scale to  $M_K$ .*

$M_W$	$SU(3) \times SU(2) \times U(1)$	quarks, gluons, leptons
	SM	gauge bosons, Higgs
OPE	$\Downarrow$	perturbative
$m_c$	$\mathcal{L}_{\text{QCD}}^{N_f=3}, \mathcal{L}_{\text{eff}}^{\Delta S=1}, \dots$	light quarks, gluons
		leptons, photon
symmetries	$\Downarrow$	nonperturbative
$M_K$	$\mathcal{L}_{\text{eff}}^{\text{SM}}$	hadrons
	(Table 2)	leptons, photon

## 1 Survey

Chiral perturbation theory (CHPT) sets in where the operator product expansion (OPE) stops (Table 1). Hadronic matrix elements of quark operators are calculated with an effective field theory directly in terms of hadron fields. The theory shares the symmetries of the effective theory of three light quarks, gluons, photon and leptons derived from the Standard Model via the OPE at a scale of the order of  $m_c$ . The step from  $m_c$  down to  $M_K$  entails an enormous loss of information that is encoded in the coupling constants (LECs) of the effective chiral Lagrangian  $\mathcal{L}_{\text{eff}}^{\text{SM}}$  in Table 2. All the Lagrangians in Table 2 are used in present-day CHPT calculations of kaon decays.

 Table 2: *Effective chiral Lagrangian  $\mathcal{L}_{\text{eff}}^{\text{SM}}$  of the SM relevant for kaon physics. The numbers in brackets denote the numbers of low-energy constants.*

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}^{\text{odd}}(0)$ $+ \mathcal{L}_{G_F p^2}^{\Delta S=1}(2)$ $+ \mathcal{L}_{e^2 p^0}^{\text{em}}(1) + \mathcal{L}_{G_8 e^2 p^0}^{\text{emweak}}(1)$ $+ \mathcal{L}_{p^4}^{\text{even}}(10) + \mathcal{L}_{p^6}^{\text{odd}}(32)$	$L = 0$
$+ \mathcal{L}_{G_8 p^4}^{\Delta S=1}(22) + \mathcal{L}_{G_{27} p^4}^{\Delta S=1}(23)$ $+ \mathcal{L}_{e^2 p^2}^{\text{em}}(14) + \mathcal{L}_{G_8 e^2 p^2}^{\text{emweak}}(15) + \mathcal{L}_{e^2 p}^{\text{leptons}}(4)$	$L = 1$
$+ \mathcal{L}_{p^6}^{\text{even}}(90)$	$L = 2$

The OPE leads to the standard classification of  $K$  decays: nonleptonic decays correspond

to four-quark operators and semileptonic decays are governed by matrix elements of mixed quark-lepton operators. For the actual decays, this correspondence is however not one-to-one. The standard classification applies directly to decays of the type  $K \rightarrow n\pi$  (nonleptonic;  $\mathcal{L}_{G_F p^n}^{\Delta S=1}$ ) and  $K \rightarrow n\pi + W^*(\rightarrow l\nu_l)$  (semileptonic;  $\mathcal{L}_{p^n}$ ). On the other hand, decays of the type  $K \rightarrow n\pi + \bar{l}l$  (with  $l$  either a charged lepton or a neutrino) make up an additional class. In this case, both semileptonic and nonleptonic operators may contribute via their respective effective Lagrangians. To illustrate this class of decays, I consider two prominent examples.

For the decays  $K \rightarrow \pi\nu\bar{\nu}$ , the nonleptonic decay chain  $K \rightarrow \pi Z^*(\rightarrow \nu\bar{\nu})$  induced by  $\mathcal{L}_{G_F p^n}^{\Delta S=1}$  is strongly suppressed: these decays are almost exclusively given by matrix elements of semileptonic operators and are therefore classified as short-distance dominated. The situation is different for the decays  $K \rightarrow \pi l^+ l^-$  where the nonleptonic mechanism  $K \rightarrow \pi\gamma^*(\rightarrow \bar{l}l)$  is important (long-distance dominance). Matrix elements of the corresponding semileptonic operators are only relevant for CP violation.

### Current CHPT activities in kaon physics<sup>1</sup>

- Systematic higher-order calculations including two-loop amplitudes for semileptonic decays ( $\rightarrow$  Sec. 2).
- Dispersion theoretic methods for nonleptonic decays ( $\rightarrow$  Colangelo, Paschos).
- Methods for determining low-energy constants ( $\rightarrow$  D'Ambrosio, de Rafael).
- CHPT and lattice ( $\rightarrow$  Golterman).
- Systematic inclusion of isospin violation and electromagnetic corrections, both for semileptonic ( $\rightarrow$  Sec. 3) and nonleptonic decays ( $\rightarrow$  Donoghue, Gardner).

Although I will concentrate in the following on semileptonic decays, I want to emphasize the importance of isospin violation and electromagnetic corrections for the dominant (nonleptonic) decays  $K \rightarrow 2\pi, 3\pi$ . Isospin breaking is estimated to be of the following size in  $K$  decays:

$O(m_u - m_d)$	$O(\alpha)$
$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2} \sim 1.5\%$	$\frac{M_{\pi^+}^2 - M_{\pi^0}^2}{M_K^2} \sim 0.5\%$

The effects of  $O(m_u - m_d)$  and  $O(\alpha)$  are comparable in size and they are small. However, there is a possible strong enhancement in the subdominant  $\Delta I = 3/2$  amplitudes because of the  $\Delta I = 1/2$  rule, e.g., in the  $K \rightarrow \pi\pi$  amplitude  $A_2$ :

$$A_2^{\text{ind}}/A_2 \sim \frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2} \cdot \frac{A_0}{A_2} \sim 0.35. \quad (1)$$

Here, the standard ratio  $A_0/A_2 \simeq 22$  (assuming isospin conservation) is used.

In this connection, I want to comment on a recent reanalysis of  $K \rightarrow 2\pi, 3\pi$  by Cheshkov [1] on the basis of the CHPT amplitudes to  $O(p^4)$  [2]. Both the (experimental) isospin amplitudes and slope parameters and the corresponding CHPT quantities assume isospin conservation. Therefore, the observed impressive agreement [1] between theory and experiment may be somewhat elusive and certainly does not imply that isospin violation is negligible. On the contrary, the neglect of isospin breaking probably hides interesting physics in the subdominant  $\Delta I = 3/2$  quantities.

---

<sup>1</sup>For lack of space, references to the original work are omitted here; instead, I refer to corresponding contributions to these Proceedings.

## 2 Semileptonic decays: $K_{l4}$ and $\pi\pi$ scattering

### 2.1 $\pi\pi$ scattering

The final state interaction of the two pions in  $K_{e4}$  decays allows for the extraction of the phase shift difference  $\delta_0^0(s) - \delta_1^1(s)$ . Recent theoretical work and new experimental results have led to major improvements.

The CHPT amplitudes to  $O(p^6)$  have been known for some time [3]. The more recent dispersion theoretic analysis [4] (Roy equations) is a priori completely independent of QCD. With high-energy ( $\sqrt{s} \geq 0.8$  GeV) scattering data as input, it yields  $S$  and  $P$  waves at low energies in terms of only two subtraction constants that can be chosen as the  $S$ -wave scattering lengths  $a_0^0, a_0^2$ . The phase shifts ( $l \leq 1$ ) and the remaining threshold parameters are then predicted [4] with amazing accuracy in terms of  $a_0^0, a_0^2$ .

In the next step, Colangelo et al. [5] matched the Roy solutions to the CHPT amplitudes. With some additional input (most importantly, the LECs  $l_3, l_4$  of  $O(p^4)$ ), the  $S$ -wave scattering lengths were determined as

$$a_0^0 = 0.220 \pm 0.005 \quad (2)$$

$$a_0^2 = -0.0444 \pm 0.0010, \quad (3)$$

fixing in turn the phase shifts at low energies. Comparing successive orders, one finds that the chiral expansion of the  $\pi\pi$  amplitude “converges” well. The weak link in this argument was the standard CHPT value for  $l_3$  that is not beyond discussion. To close this loophole,  $l_3$  was taken as a free parameter to be determined [6] from a fit to the new data from BNL-E865 [7]. Including the old data of the Geneva-Saclay experiment [8], the best fit value for  $a_0^0$  was found to be  $a_0^0 = 0.221$ , in complete agreement with (2). The dependence on  $a_0^0$  is shown in Fig. 1 together with the available experimental results for the phase shift difference. This agreement in turn corroborates the usual value of  $l_3$  and the standard low-energy expansion scheme corresponding to a large quark condensate. At least for chiral  $SU(2)$ , the motivation for a generalized scheme [9] with a substantially smaller quark condensate has evaporated.

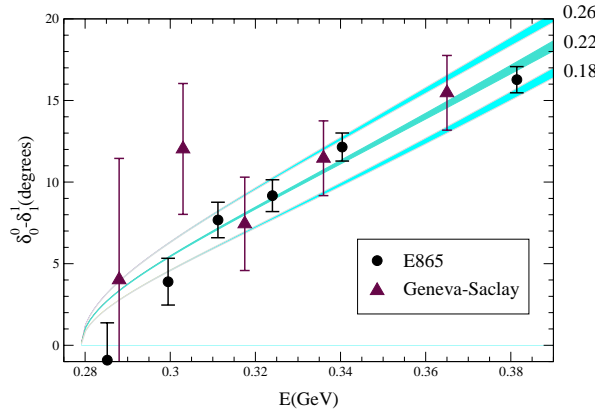


Figure 1: *Phase shift difference  $\delta_0^0 - \delta_1^1$  for different values of  $a_0^0$  [6].*

With the level of accuracy reached (both in CHPT and in the dispersive analysis), isospin breaking must be included. This is partly available for  $\pi\pi$  scattering [10] but not yet for the actual  $K_{e4}$  decays.

## 2.2 $K_{e4}$ form factors

Whereas the vector form factor  $H$  is dominated by the leading contribution of  $O(p^4)$  due to the chiral anomaly, there are substantial corrections [11] of  $O(p^6)$  to the axial form factors  $F, G$ . The impact of those corrections can be seen in the updated values [11] of some of the LECs of  $O(p^4)$  collected in Table 3. Although the errors in the last column do not include theoretical uncertainties it is clear that the  $O(p^6)$  corrections induce sizable shifts in some of the LECs ( $L_2, L_5, L_8$ ).

Table 3: *Phenomenological values of  $L_i^r(M_\rho) \times 10^3$ . The new values in the last column are from the work of Amorós et al. [11].*

i	1995	ABT 2001
1	$0.4 \pm 0.3$	$0.43 \pm 0.12$
2	$1.35 \pm 0.3$	$0.73 \pm 0.12$
3	$-3.5 \pm 1.1$	$-2.35 \pm 0.37$
5	$1.4 \pm 0.5$	$0.97 \pm 0.11$
7	$-0.4 \pm 0.2$	$-0.31 \pm 0.14$
8	$0.9 \pm 0.3$	$0.60 \pm 0.18$

## 3 Isospin violation and electromagnetic corrections in $K_{l3}$ decays

The most precise determination of the CKM matrix element  $V_{us}$  comes from  $K_{e3}$  decays. The present status [12] indicates a possible problem ( $2.2 \sigma$ ) for three-generation mixing:

	$ V_{us} $
$K_{l3}$	$0.2196 \pm 0.0023$
$ V_{ud}  + \text{unitarity}$	$0.2287 \pm 0.0034$

Isospin breaking is an essential ingredient [13] for the extraction of  $V_{us}$ . However, a complete calculation to  $O(p^4, (m_u - m_d)p^2, e^2 p^2)$  has been undertaken only recently [14]. The relevant Lagrangians of Table 2 are  $\mathcal{L}_{p^n}(n \leq 4)$  and  $\mathcal{L}_{e^2 p}^{\text{leptons}}$  [15]. The calculation of isospin conserving corrections of  $O(p^6)$  is also under way [16].

Let me concentrate here on the radiative corrections for  $K_{l3}^+$  decays [14]. For  $\alpha = 0$ , the decay distribution takes the form

$$A_1^{(0)}(t, u)f_+(t)^2 + A_2^{(0)}(t, u)f_+(t)f_-(t) + A_3^{(0)}(t, u)f_-(t)^2 \quad (4)$$

with kinematical functions  $A_i^{(0)}(t, u)$  and form factors  $f_\pm(t)$  in terms of the Dalitz variables  $t = (p_K - p_\pi)^2$ ,  $u = (p_K - p_l)^2$ .

Radiative corrections involve the diagrams of Fig. 2 and Bremsstrahlung of soft photons. The final result is a decay distribution of the same form as (4) with

$$f_\pm(t) \longrightarrow f_\pm(t, u) \quad (5)$$

$$A_i^{(0)}(t, u) \longrightarrow A_i(t, u). \quad (6)$$

The structure dependent corrections involving the interplay of QCD and QED are contained in  $f_{\pm}(t, u)$  whereas the universal QED corrections (Coulomb part of loop corrections + Bremsstrahlung) appear in the modified kinematical functions  $A_i(t, u)$ . The latter depend of course on the experimental conditions.

Using this representation [14], the form factors  $f_{\pm}(t, u)$  can be extracted from the experimental data in a model independent way. This will allow for a more reliable determination of the form factors ( $f_-$  is still poorly known) and also, as a consequence, of the CKM matrix element  $V_{us}$ . Numerical results will be available [14] when these Proceedings appear in print.

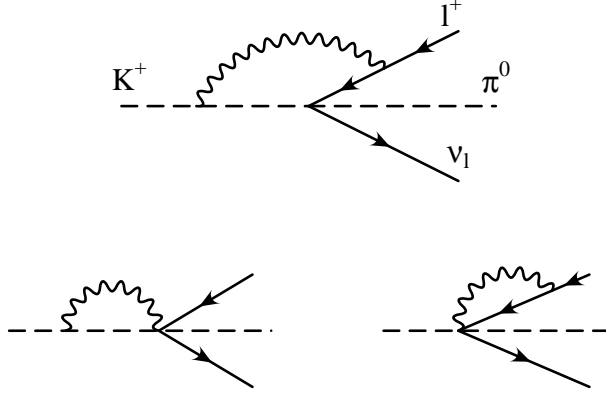


Figure 2: *Radiative corrections for  $K_{l3}^+$  decays.*

## 4 Conclusions

The combination of OPE and CHPT provides a comprehensive framework for the analysis of **all**  $K$  decays:

- The structure and the renormalization of  $\mathcal{L}_{\text{eff}}^{\text{SM}}$  are well understood including electromagnetic corrections.
- Significant advances have been achieved in the analysis of  $\pi\pi$  scattering (related to  $K_{e4}$  decays).
- $O(p^6)$ , isospin breaking and electromagnetic corrections are being completed for semileptonic decays.
- More work is needed to determine many of the LECs in order to make the scheme even more predictive.

## 5 Acknowledgements

I thank Vincenzo Cirigliano and Helmut Neufeld for information about their unpublished work [14].

# References

- [1] C. Cheshkov, hep-ph/0105131.
- [2] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. **B346**, 17 (1990); Phys. Lett. **B261**, 496 (1991).
- [3] J. Bijnens et al., Phys. Lett. **B374**, 210 (1996); Nucl. Phys. **B508**, 263 (1997).
- [4] B. Ananthanarayan et al., hep-ph/0005297.
- [5] G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. **B603**, 125 (2001).
- [6] G. Colangelo, J. Gasser and H. Leutwyler, Phys. Rev. Lett. **86**, 5008 (2001).
- [7] S. Pislak et al. (BNL-E865), hep-ex/0106071; M. Zeller, these Proceedings.
- [8] L. Rosselet et al. (Geneva-Saclay), Phys. Rev. **D15**, 574 (1977).
- [9] J. Stern, Nucl. Phys. Proc. Suppl. **64**, 232 (1998) and Refs. therein.
- [10] M. Knecht and R. Urech, Nucl. Phys. **B519**, 329 (1998); U.-G. Meißner, G. Müller and S. Steininger, Phys. Lett. **B406**, 154 (1997) and Err. **B407**, 454 (1997).
- [11] G. Amoros, J. Bijnens and P. Talavera, Nucl. Phys. **B585**, 293 (2000) and Err. **B598**, 665 (2001); Nucl. Phys. **B602**, 87 (2001).
- [12] Particle Data Group, Eur. Phys. J. **C15**, 1 (2000).
- [13] H. Leutwyler and M. Roos, Z. Phys. **C25**, 91 (1984).
- [14] V. Cirigliano, H. Neufeld et al., in preparation.
- [15] M. Knecht et al., Eur. Phys. J. **C12**, 469 (2000).
- [16] J. Bijnens et al., in preparation.